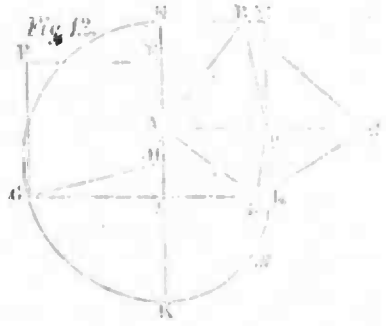
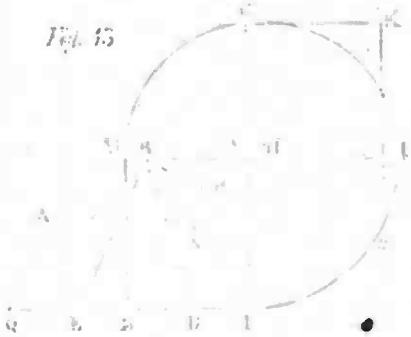
GEOMETRY OF THE SQUARE.





given figure ABCD; hence the thing required by the problem has been effected.

Demonstration.—Because the straight line NK, which is equal to the sum of the sides NI and IL of the parallelogram NILM, is divided equally in H and unequally in I; the rectangle under the two unequal parts, NI and IK, together with the square of HI, is equal to the square of HG; but the square of HG is equal to the squares of IG and HI; therefore the rectangle under NI and IK together with the square of HI, is equal to the squares of IG and HI; take away the square of HI which is common, and there remains the square of IG equal to the rectangle under NI and IK; but IK is equal to IL; therefore the square of IG is equal to the rectangle under NI and IL, which has been shewn to be equal to the given figure ABCD; consequently the square IGFE is also equal to ABCD; hence the truth of the construction is manifest.

PROBLEM.—*To construct a square that shall be equal in area to a given five-sided figure; that is, to convert a given five-sided figure into a square of equal area.*

Example:—Let ABCDE (fig. 13), be the given five-sided figure, of which the containing sides are AB, BC, CD, DE and AE; it is required to convert the five-sided figure, ABCDE, into a square of equal area.

Construction.—Draw the diagonal BE, joining the opposite angles of the figures at B and E, and dividing the figure into the triangle ABE, and the trapezium or four-sided figure, BCDE. Draw also the diagonal, BD, joining the opposite angles B and D, and dividing the trapezium, BCDE, into the two triangles, BCD and BDE, so that the whole figure is divided by the diagonals BD and BE into the three triangles, BCD, BAE, and BDE.

Through the angles A and C, which are respectively subtended by the diagonals BE and BD, and parallel to these diagonals draw the straight lines AQ and CP, meeting the side DE, produced both ways in Q and P, and draw BQ, BP intersecting the sides of the figure AE and CD in *r* and *s*; then is the triangle PBQ equal to the given five-sided figure, ABCDE.

Bisect PQ, the base of the derived triangle PBQ, in the point L, and through the vertex of the figure at B, draw the straight line BI parallel to the base PQ, which extend both ways as far as necessary; then at the points L and P, the one being the middle point, and the other the extremity, of the base PQ, erect the perpendiculars LM and PN, meeting BI, and its production on the left of the vertex B, in the points M and N, thereby forming the rectangular parallelogram PNML, which is equal in magnitude to the triangle PBQ, and, consequently, to the given five-sided figure ABCDE.

Upon N as a centre, with NP as a radius, describe the quadrant of a circle Pml, meeting BI in I, and making NI equal to NP, the side of the rectangle PNML, so that MI is equal to the sum of the sides NM and NP extended out in the same straight line. Bisect MI the sum of the sides in H, and on H as a centre, with a radius equal to HI or HM, describe the semi-circle MGI, meeting PN, the side of the rectangle produced in G; then is NG the side of a square which contains the same area as the given five-sided figure ABCDE.

Upon GN, the side of the square just found, construct the square GKFN, and the thing required is accomplished, for the five-sided figure ABCDE, the four-sided figure BCPQ, the triangle PBQ, the rectangular parallelogram PNML, and the square GKFN, are all equal, being successively derived from each other, and consequently, the figure proposed has been converted into a square of equal area.

Demonstration.—Because the straight line AQ is parallel to the diagonal BE which joins the opposite angles B and E, the triangles ABQ and AEQ constituted on the same base AQ are equal between themselves, for triangles upon the same base and between the same parallels are equal; from these equals, take away the triangle A*r*Q, which is common to them both, and the remainder AB*r* is equal to the remainder QE*r*, so that the part AB*r*, which is cut off from the given figure by the straight line BQ, is equal to the part QE*r*, which is applied to it by producing the side DE to meet BQ in Q, and consequently, the five-sided figure ABCDE originally proposed is equal in magnitude to the derived figure of four sides BCDQ.

In like manner, because the straight line CP is parallel to the diagonal BD which joins the opposite angles B and D, the triangles CBP and CDP constituted on the same base CP are equal between themselves, for triangles upon the same base and between the same parallels are equal; from these equals take away the triangle C*s*P, which is common to them both, and the remainder B*s*C is equal to the remainder D*s*P; that is, the part B*s*C, which is cut off from the given figure by the straight line BP, is equal to the part D*s*P, which is applied to it by producing the side ED to meet the straight line BP in P, and consequently, the whole triangle PBQ thus formed, is equal to the four-sided figure BCDQ, which has been shewn to be equal to the five-sided figure, ABCDE, originally proposed; therefore, the triangle PBQ is also equal to the given five-sided figure, for things that are equal to the same thing are equal to one another.

Again, the rectangular parallelogram PNML is equal to the triangle PBQ, since it is constituted upon half the base PL, and is situated between the same parallels MI and PQ; but the triangle PBQ is equal to the proposed five-sided figure ABCDE; hence, also, the rectangular parallelogram PNML is equal to the given figure, for things that are equal to the same thing are equal to one another.

Lastly, the square GKFN is likewise equal to the given figure ABCDE, for it is equal to the rectangle or product under NM and NI, the sides of the rectangle PNML, NI being by construction equal to NP; but the rectangle PNML has been shewn to be equal to the proposed figure ABCDE, and for this reason, the square GKFN is also equal to it, for things that are equal to the same thing are equal to one another; consequently, the given five-sided figure has been converted into a square of equal area as required by the problem.

ON THE GENERAL SIZE OF STONES IN NORMAN ARCHITECTURE.*

An architect of the present day may make a design which may be an accurate imitation of the ancient Norman buildings, but it will be in vain that he has had recourse to the finest models, and paid attention to the most correct details of each separate moulding, ornament, &c., if he has neglected to give his builder strict injunctions as to the *size* of the blocks of stone that are employed. The improvements in modern masonry have enabled us to introduce larger blocks of stones than were used by the Normans; and the effect of this (even if it be allowed to be an improvement) is, that any person, though he may be a stranger to the rules of architecture, will at once feel that the building looks modern.

I have myself found this to be the case in a chapel for which I gave a design in my own neighbourhood. During my absence, the mason selected his finest blocks of stone, and instead of making each zig-zag of the arches a separate stone, four or five large ones have been used, by which the eye is at once convinced that it is not correctly *ancient* Norman, though the reason perhaps few would discover.

The south transept of Norwich cathedral has lately been repaired, and it is very well executed; but the depth of the stones being about 17 or 18 inches (nearly double the size of those of the old Norman buildings), the new work has a modern appearance.

I subjoin a list of the size of the stones which I have measured in many buildings, as it may better impress upon the memory a subject which I think is one of considerable architectural importance. In all the following buildings, the stones are regularly placed in horizontal courses, although each course varies in depth from six and a half to nine inches, but a very few may occasionally be found an inch more or less.

The general size of the stones in Norwich cathedral are:—

1. At the west end of the nave, from 6½ to 9 inches deep.
2. The upper part of the transept about 8 inches.
3. The centre tower (the upper part), 7 or 8 inches.†
4. St. Luke's chapel, 6½ to 8 inches; average, 7½.
5. The dormitory (near the cathedral), from 6 to 7 inches; a very few, 7½ or 8 inches.
6. Norwich castle, about 7 or 8 inches.‡
7. The buttresses to Castle Acre, 6 to 7½ or 8 inches.
8. Entrance gateway to ditto, 6½ to 7½ inches; a few 9 inches.
9. Castle Acre Priory. The nave at the west end, 7½ to 9 inches.
10. Ditto. Centre tower, 8½ to 9 inches.
11. St. Edmund's Bury. The Norman gate way, 7 to 9 inches.
12. Ditto. The Norman building in the Market-place, 7 to 8½ inches.

* From the current No. of "The Journal of the British Archæological Association."

† The lower part of the tower was repaired more than fifty years ago with larger stones.

‡ The lower part of the south side has lately been repaired. Its regular joints each stone being 5 inches deep, which is not like that in the ancient Norman work.

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